

# MATHEMATICS

## 2024 UTME MODEL QUESTIONS MATHEMATICS

1. If  $x$  represents the median and  $y$  the mode of the measurements 5,9,3,5,8, then  $(x, y)$  is (A) 5,8 (B) 6,5 (C) 5,5 (D) 7,5
2. Olu sold his bicycle to Ade at a profit of 10%. Ade sold it to Ola for #209 at a loss of 5%. How much did the bicycle cost Olu? (A) #200 (B) #180 (C) #196 (D) #410
3. Evaluate  $212_3 - 121_3 + 222_3$  (A)  $313_3$  (B)  $1000_3$  (C)  $1020_3$  (D)  $1222_3$
4. If the length of a square is increased by 20% while its width is decreased by 20% to form a rectangle, what is the ratio of the area of the rectangle to the area of the square? (A) 6: 5 (B) 25: 24 (C) 5: 6 (D) 24: 25
5. If  $\log_{10} 2 = 0.3010$  and  $\log_{10} 3 = 0.4771$ , evaluate without using logarithm tables  $\log_{10} 4.5$  (A) 0.9542 (B) 0.6532 (C) 0.4771 (D) 0.3010
6. Four interior angles of a pentagon are  $(90 - x)^\circ$ ,  $(90 + x)^\circ$ ,  $(110 - 2x)^\circ$  and  $(110 + 2x)^\circ$ , find the fifth interior angle (A)  $110^\circ$  (B)  $120^\circ$  (C)  $130^\circ$  (D)  $140^\circ$
7. If  $a : B = 5 : 8$ ,  $x : y = 25 : 16$ , evaluate  $\frac{a}{x} : \frac{b}{y}$  (A) 125: 128 (B) 3: 5 (C) 3: 4 (D) 2: 5
8. If  $-8, m, n, 19$  are in arithmetic progression, find  $(m, n)$  (A) 1,10 (B) 2,10 (C) 3,13 (D) 4,16
9. Simplify  $\sqrt{160x^2 + \sqrt{71x^4 + \sqrt{100x^8}}}$  (A)  $9x^2$  (B)  $12\sqrt{3x}$  (C)  $13x$  (D)  $\sqrt{13x}$
10. Rationalize  $\frac{2\sqrt{3}+3\sqrt{2}}{3\sqrt{2}-2\sqrt{3}}$  (A)  $5 - 2\sqrt{6}$  (B)  $5 + 2\sqrt{6}$  (C)  $5\sqrt{3}$  (D)  $5\sqrt{6} + 2$
11. Find  $n$  if  $34_n = 10011_2$  (A) 5 (B) 6 (C) 7 (D) 8
12. Express  $\frac{5x-12}{(x-2)(x-3)}$  in partial fractions (A)  $\frac{2}{x+2} - \frac{3}{x-3}$  (B)  $\frac{2}{x-2} + \frac{3}{x-3}$  (C)  $\frac{2}{x-3} - \frac{3}{x-2}$  (D)  $\frac{5}{x-3} + \frac{4}{x-2}$
13. Two variables  $x$  and  $y$  are such that  $\frac{dy}{dx} = 4x - 3$  and  $y = 5$  when  $x = 2$   
Find  $y$  in terms of  $x$  (A)  $2x^2 - 3x + 5$  (B)  $2x^2 - 3x + 2$  (C)  $2x^2 - 3x + 3$  (D)  $3x^2 - 2x + 3$
14. Determine  $x + y$  if  $\begin{pmatrix} 2 & -3 \\ -1 & 4 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -1 \\ 8 \end{pmatrix}$  (A) 3 (B) 4 (C) 1 (D) 7
15. The angle of elevation of the top of a building from a measuring instrument placed on the ground is  $30^\circ$ . If the

- ca building is 40m high, how far is the instrument from the foot  
of of the building? (A)  $20/\sqrt{3}m$  (B)  $40/\sqrt{3}m$  (C)  $20\sqrt{3}m$  (D)  
40 $\sqrt{3}m$
16. A number is selected at random between 10 and 20, both  
ia numbers inclusive. Find the probability that the number is an  
even number (A)  $\frac{5}{11}$  (B)  $\frac{1}{2}$  (C)  $\frac{6}{11}$  (D)  $\frac{4}{11}$
17. The first term of a geometric progression is twice its common  
ratio. Find the sum of the first two terms of the progression if  
its sum to infinity is 8 (A)  $\frac{8}{5}$  (B)  $\frac{8}{3}$  (C)  $\frac{72}{25}$  (D)  $\frac{25}{72}$
18. A straight line makes an angle of  $30^\circ$  with the positive  
 $x$ -axis and cuts the  $y$ -axis at  $y = 5$ . Find the equation of the  
line. (A)  $\sqrt{3}y = x + 5\sqrt{3}$  (B)  $\sqrt{3}y = -x + 5\sqrt{3}$  (C)  $y = x +$   
5 (D)  $\frac{1}{10}x + 5$
19. Evaluate  $\int \sin 3x dx$  (A)  $\frac{2}{3} \cos 3x + C$  (B)  $\frac{1}{3} \cos 3x + C$  (C)  
5  $-\frac{1}{3} \cos 3x + C$  (D)  $-\frac{2}{3} \cos 3x + C$
20. The probability that a student passes any examination is  $\frac{2}{3}$ . If  
the student takes three examinations, what is the probability  
that he will not pass any of them? (A)  $2/3$  (B)  $\frac{4}{9}$  (C)  $\frac{8}{27}$  (D)  $\frac{1}{27}$
21. How many three-digit numbers can be formed from 32564  
without any digit being repeated? (A) 12 (B) 10 (C) 120 (D)  
60
22. Calculate the mean deviation of the set of numbers  
7,3,14,9,7 and 8 (A)  $2\frac{1}{6}$  (B)  $2\frac{1}{2}$  (C)  $1\frac{1}{6}$  (D)  $2\frac{1}{3}$
23. In a class of 40 students, 32 offer mathematics, 24 offer  
physics. How many offer both Mathematics and Physics? (A)  
20 (B) 16 (C) 8 (D) 4
24. If  $\frac{9^{2x-1}}{27^{x+1}} = 1$ , find the value of  $x$  (A) 8 (B) 5 (C) 3 (D) 2
25. If  $\begin{vmatrix} -x & 2 \\ 4x & 1 \end{vmatrix} = \begin{vmatrix} 3 & 3x \\ 4 & -5 \end{vmatrix}$ , find the value of  $x$  (A) 5 (B) 2 (C) -2  
(D) 5
26. The length a person can jump is inversely proportional to his  
weight. If a 20kg person can jump 1.5m, find the constant of  
proportionality (A) 15 (B) 20 (C) 30 (D) 60
27. An arc of a circle subtends an angle of  $30^\circ$  on the  
circumference of a circle of radius 21cm. Find the length of  
an arc (A) 11cm (B) 22cm (C) 44cm (D) 88cm
28. A triangle has vertices  $P(-1,6)$ ,  $Q(-3,-4)$  and  $R(1,-4)$ .  
Find the midpoints of  $PQ$  and  $QR$  respectively (A)  $(0,-2)$   
and  $(-1,-4)$  (B)  $(-1,0)$  and  $(-1,-1)$  (C)  $(-2,1)$  and  $(0,1)$   
(D)  $(-2,1)$  and  $(-1,-4)$
29. What is the rate of change of the volume  $V$  of a hemisphere  
with respect to its radius  $r$  when  $r = 2$ ? (A)  $2\pi$  (B)  $16\pi$  (C)  
 $8\pi$  (D)  $4\pi$
30. What is the quotient when  $6x^2 - 13x + 5$  is divided by  $2x -$   
1? (A)  $3x + 5$  (B)  $3x - 5$  (C)  $5x - 3$  (D)  $5x + 3$
31. If  $\sin \theta = -\frac{1}{2}$  for  $0 < \theta < 360^\circ$ , the value of  $\theta$  is (A)  $30^\circ$   
and  $150^\circ$  (B)  $150^\circ$  and  $330^\circ$  (C)  $210^\circ$  and  $330^\circ$  (D)  $150^\circ$   
and  $210^\circ$
32. For what value of  $n$  is  ${}^{n+1}C_3 = 4({}^nC_3)$  (A) 6 (B) 5 (C) 4 (D)  
3
33. A binary operation  $*$  on the set of rational numbers is defined  
as  $x * y = \frac{x^2 - y^2}{2xy}$  find  $-5 * 3$  (A)  $-\frac{8}{15}$  (B)  $\frac{8}{15}$  (C)  $\frac{17}{15}$  (D)  $-\frac{17}{15}$



34. Find the value of  $\frac{\tan 60^\circ - \tan 30^\circ}{\tan 60^\circ + \tan 30^\circ}$  (A)  $4/\sqrt{3}$  (B)  $2/\sqrt{3}$  (C)  $\frac{1}{2}$  (D) 1
35. The area of a square is  $144\text{cm}^2$ . Find the length of the diagonal (A) 13cm (B)  $12\sqrt{2}\text{cm}$  (C) 12cm (D)  $11\sqrt{3}\text{cm}$
36. If  $\log_{x^{1/2}} 64 = 3$ , find the value of  $x$  (a) 4 (B) 16 (C) 32 (D) 64
37. Factorize completely  $(4x + 3y)^2 - (3x - 2y)^2$  (A)  $(x + 5y)(7x + y)$  (B)  $(x + 5y)(7x - y)$  (C)  $(x - 5y)(7x + y)$  (D)  $(x - 5y)(7x - y)$
38. The fifth term of an A.P is 24 and the eleventh term is 96. Find the first term (A) 12 (B) 4 (C) -12 (D) -24
39. Find the remainder when  $2x^3 - 11x^2 + 8x - 1$  is divided by  $x + 3$  (A) -871 (B) -781 (C) -187 (D) -178
40. The probabilities that a man and his wife live for 80 years are  $\frac{2}{3}$  and  $\frac{3}{5}$  respectively. Find the probability that only one of them will live up to 80 years (A)  $\frac{4}{15}$  (B)  $\frac{7}{15}$  (C)  $\frac{3}{15}$  (D)  $\frac{1}{15}$

**SOLUTION**

1. **C:** - Mode is the value with the highest frequency. Thus,  $y = 5$   
 - To get the median, we arrange the data in order, we then identify the middle value (the median)

The numbers in ascending order are:

$$3, 5, \boxed{5}, 8, 9$$

Then  $x = 5$

So,  $(x, y) = (5, 5)$

2. **A:** Let the cost of price be  $x$   
 Amount Olu sold it = 10% of  $x + x$   
 (Since a profit of 10% was made)  
 $= 0.1x + x = 1.1x$

Amount Ade sold it =  $1.1x - 5\%$  of  $1.1x$  (Since Ade incurred a loss of 5%)

$$= 1.1x - 0.055x = 1.045x$$

Since the actual amount Ade sold was 209, we have:

$$1.045x = \text{\#}209$$

$$x = \frac{\text{\#}209}{1.045} = \text{\#}200$$

3. **C:** Method 1

$$\begin{array}{r} 212_3 \\ + 222_3 \\ - \underline{121_3} \\ \hline 1020_3 \end{array}$$

**Method 2**

We can convert each value to its decimal equivalent, perform the operation in decimal and then return the result to base 3

$$210$$

$$212_3 = 2 \times 3^2 + 1 \times 3^1 + 2 \times 3^0 = 23_{ten}$$

$$210$$

$$121_3 = 1 \times 3^2 + 2 \times 3^1 + 1 \times 3^0 = 16_{ten}$$

$$210$$

$$222_3 = 2 \times 3^2 + 2 \times 3^1 + 2 \times 3^0 = 26_{ten}$$

Then, we have  $212_3 - 121_3 + 222_3 = 23_{ten} = 23_{ten}$

$$16_{ten} + 26_{ten} = 33_{ten}$$

We now convert  $33_{ten}$  to base 3

$$\begin{array}{l|l} 3 & 33 \\ 3 & 11r0 \\ 3 & 3r2 \\ 3 & 1r0 \\ 3 & 0r1 \end{array} \uparrow$$

Thus,  $212_3 - 121_3 + 222_3 = 1020_{\text{ten}}$

4. **D:** Let each side of the square be  $x$

Then, length of the rectangle =  $x + 20\%$  of  $x$

$$x + 0.2x = 1.2x$$

Breadth of the rectangle =  $x - 20\%$  of  $x$

$$= x - 0.2x = 0.8x$$

$$\text{Area of square} = x^2$$

$$\text{Area of the rectangle} = 1.2x \times 0.8x = 0.96x^2$$

$$\frac{\text{Area of rectangle}}{\text{Area of square}} = \frac{0.96x^2}{x^2} = 0.96$$

$$= \frac{96}{100} = \frac{24}{25}$$

$$\text{Thus, } A_{\text{rectangle}} : A_{\text{square}} = 24 : 25$$

5. **B:**  $\log_{10} 45 = \log_{10} \left(\frac{9}{2}\right)$

$$= \log_{10} 9 - \log_{10} 2$$

$$= \log_{10} 3^2 - \log 2$$

$$= 2 \log_{10} 3 - \log 2$$

$$= 2(0.4771) - 0.3010 = 0.6532$$

6. **D:** sum of the interior angles of a polygon with  $n$  sides =  $(n - 2)180^\circ$

For a pentagon,  $n = 5$ . Then,

$$\text{Sum} = (5 - 2)180^\circ = 3 \times 180^\circ = 540^\circ$$

Let the fifth angle be  $y$

$$90 - x + 90 + x - 110 - 2x + y = 540$$

$$400 + y = 540$$

$$y = (540 - 400)^\circ = 140^\circ$$



7. **D:**  $a:b = 5:8$  and  $x:y = 25:16$ . Then,

$$\frac{a}{b} = \frac{5}{8}; 8a = 5b \dots (i)$$

$$\frac{x}{y} = \frac{25}{16}; 16x = 25y \dots (ii)$$

divide (i) by (ii)

$$\frac{8a}{16x} = \frac{5b}{25y}$$

$$\left(\frac{8}{16}\right) \left(\frac{a}{x}\right) = \left(\frac{5}{25}\right) \left(\frac{b}{y}\right)$$

$$\frac{a}{x} \div \frac{b}{y} = \frac{5}{25} \div \frac{8}{16}$$

$$= \frac{1}{5} \div \frac{1}{2} = \frac{1}{5} \times \frac{2}{1} = \frac{2}{5}$$

$$\text{Thus, } \frac{a}{x} : \frac{b}{y} = 2:5$$

8. **A:** Given:  $-8, m, n, 19$  as an AP

$$T_4 = a + 3d \text{ (since } T_n = a + (n - 1)d)$$

$$19 = -8 + 3d$$

$$3d = 19 + 8 = 27, d = \frac{27}{3} = 9$$

$$m = T_2 = a + d = -8 + 9 = 1$$

$$n = T_3 = a + 2d = -8 + 2(9) = 10$$

$$\text{Then, } (m, n) = (1, 10)$$

9. **C:**  $\sqrt{160x^2 + \sqrt{71x^4 + \sqrt{100x^8}}}$

$$= \sqrt{160x^2 + \sqrt{81x^4}}$$

$$= \sqrt{160x^2 + \sqrt{9x^4}}$$

$$= \sqrt{169x^2}$$

$$= (13^2 x^2)^{1/2}$$

$$= [(13x)^2]^{1/2}$$

$$= 13x$$

$$10. \frac{2\sqrt{3}+3\sqrt{2}}{3\sqrt{2}-2\sqrt{3}} = \frac{2\sqrt{3}+3\sqrt{2}}{3\sqrt{2}-2\sqrt{3}} \times \frac{3\sqrt{2}+2\sqrt{3}}{3\sqrt{2}+2\sqrt{3}}$$

$$= \frac{6\sqrt{6} + (4 \times 3) + (9 \times 2) + 16\sqrt{6}}{(3\sqrt{2})^2 - (2\sqrt{3})^2}$$

$$= \frac{12\sqrt{6} + 30}{18 - 12}$$

$$= \frac{12\sqrt{6} + 30}{6}$$

$$= \frac{6[2\sqrt{6} + 5]}{6}$$

$$= 2\sqrt{6} + 5$$

11. A:

1 0 4 3 2 1 0

$$34_n = 10011_2 = 1 \times 2^4 + 0 \times 2^3 + 0 \times 2^2 + 1 \times 2^1 + 1 \times 2^0$$

$$3n + 4 = 16 + 2 + 1$$

$$3n + 4 = 19, 3n = 15, n = \frac{15}{3} = 5$$

$$12. \text{ B: } \frac{5x-12}{(x-2)(x-3)} = \frac{A}{x-2} + \frac{B}{x-3}$$

$$= \frac{A(x-3) + B(x-2)}{(x-2)(x-3)}$$

$$\text{Then, } A(x-3) + B(x-2) = 5x - 12$$

$$A(x-3) + B(x-2) = 5x - 12$$

$$\text{Let } x = 3$$

$$A(3-3) + B(3-2) = 5(3) - 12$$

$$\text{Let } x = 2$$

$$A(2-3) + B(2-2) = 5(2) - 12$$

$$-A = 10 - 12 = -2, A = 2$$

$$\text{Then, } \frac{5x-12}{(x-2)(x-3)} = \frac{2}{x-2} + \frac{3}{x-3}$$

$$13. \text{ C: } \frac{dy}{dx} = 4x - 3$$

$$y = \int \frac{dy}{dx} = \int 4x - 3$$

$$y = \frac{4x^{1+1}}{1+1} - \frac{3x^{0+1}}{0+1} + k = 2x^2 - 3x + k$$

$$\text{When } x = 2, y = 5$$

Thus, we have:

$$5 = 2(2)^2 - 3(2) + k$$

$$5 = 8 - 6 + k$$

$$5 = 2 + k, k = 3$$

$$\text{Then, } y = 2x^2 - 3x + 3$$

$$14. \text{ D: } \begin{pmatrix} 2 & -3 \\ -1 & 4 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -1 \\ 8 \end{pmatrix}$$

$$2x - 3y = -1 \dots (i)$$

$$-x + 4y = 8 \dots (ii)$$

Multiply (ii) by 2

$$-2x + 8y = 16 \dots (iii)$$

Add (i) and (iii)

$$2x + (-2x) + (-3y) + 8y = -1 + 16$$



$$5y = 15, y = \frac{15}{5} = 3$$

Put  $y = 3$  into (ii)

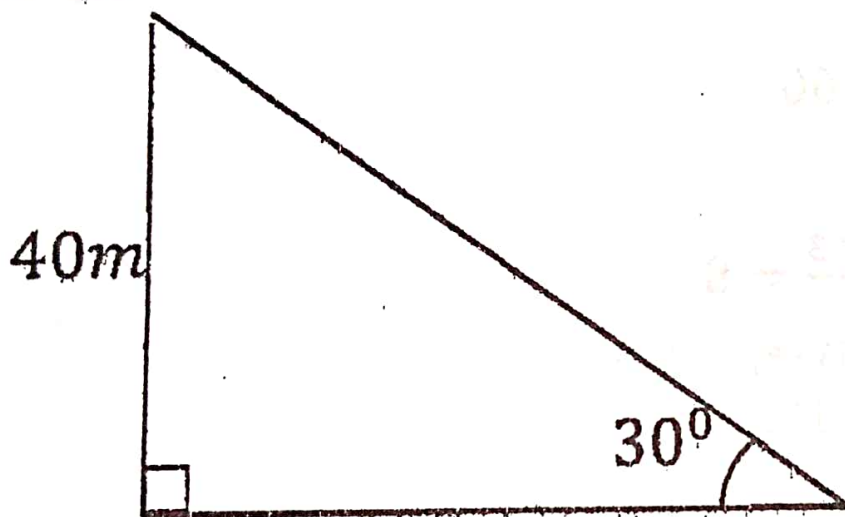
$$-x + 4(3) = 8$$

$$-x + 12 = 8$$

$$-x = 8 - 12 = -4, x = 4$$

$$\text{Then, } x + y = 4 + 3 = 7$$

15. B:



$$\tan 30^\circ = \frac{40m}{d}$$

$$d = \frac{40m}{\tan 30^\circ} = 40\sqrt{3}m$$

16. C: counting from 10 to 20, we have a total of 11 numbers.

The even ones are 10, 12, 14, 16, 18, 20 (six numbers)

Thus, the probability of selecting an even number =  $\frac{6}{11}$

17. C:  $S_{\infty} = \frac{a}{1-r}$

Since  $a = 2r$  and  $S_{\infty} = 8$ , we have:

$$8 = \frac{2r}{1-r}$$

$$8 - 8r = 2r, 8 = 10r, r = \frac{8}{10} = \frac{4}{5}$$

$$a = 2r = 2\left(\frac{4}{5}\right) = \frac{8}{5}$$

$$S_n = \frac{a(1-r^n)}{1-r}$$

$$\text{Thus, } S_2 = \frac{\frac{8}{5} \left[ 1 - \left(\frac{4}{5}\right)^2 \right]}{1 - \frac{4}{5}} = \frac{\frac{8}{5} \left[ 1 - \frac{16}{25} \right]}{\frac{1}{5}}$$

$$S_2 = \frac{\frac{8}{5} \left(\frac{9}{25}\right)}{\frac{1}{5}}$$

$$= \frac{8}{5} \times \frac{9}{25} \times \frac{5}{1} = \frac{8 \times 9}{5 \times 5} = \frac{72}{25}$$

*Method 2*

$$S_2 = a + ar$$

$$= \frac{8}{5} + \frac{8}{5} \left(\frac{4}{5}\right) = \frac{8}{5} + \frac{32}{25} = \frac{40 + 32}{25} = \frac{72}{25}$$

18. A:  $m = \tan \theta = \tan 30^\circ = 1/\sqrt{3}$ ,  $c = 5$  (given)

$$y = mx + c$$

$$y = \frac{1}{\sqrt{3}}x + \frac{5}{1} = \frac{x+5\sqrt{3}}{\sqrt{3}}$$

$\therefore y\sqrt{3} = x + 5\sqrt{3}$  (Written as  $\sqrt{3}y = x + 5\sqrt{3}$  in option A)

19. Let  $u = 3x$ ,  $\frac{du}{dx} = 3$ . Thus,  $dx = \frac{du}{3}$

$$\text{Then, } \int \sin 3x \, dx = \int \sin u \cdot \frac{du}{3}$$

$$= \frac{1}{3} \int \sin u \, du$$

$$= \frac{1}{3} [-\cos u + c]$$

$$= \frac{1}{3} [-\cos(3x) + c]$$

$$= -\frac{1}{3} \cos 3x + c$$

20. D:  $P_{pass} = \frac{2}{3}$ . Then,  $P_{fail} = 1 - \frac{2}{3} = \frac{1}{3}$

$$P_{notpassany} = P_{failallthree} = \frac{1}{3} \times \frac{1}{3} \times \frac{1}{3} = \frac{1}{27}$$

21. D: since there are 5 digits and repetition is not allowed, the number of three-digit numbers that can be formed  $= {}^5 P_3$

$$= \frac{5!}{(5-3)!}$$

$$= \frac{5!}{2!}$$

$$= \frac{5 \times 4 \times 3 \times 2 \times 1}{2!} = 60$$

22. D: Given: 7, 3, 14, 9, 7, 8

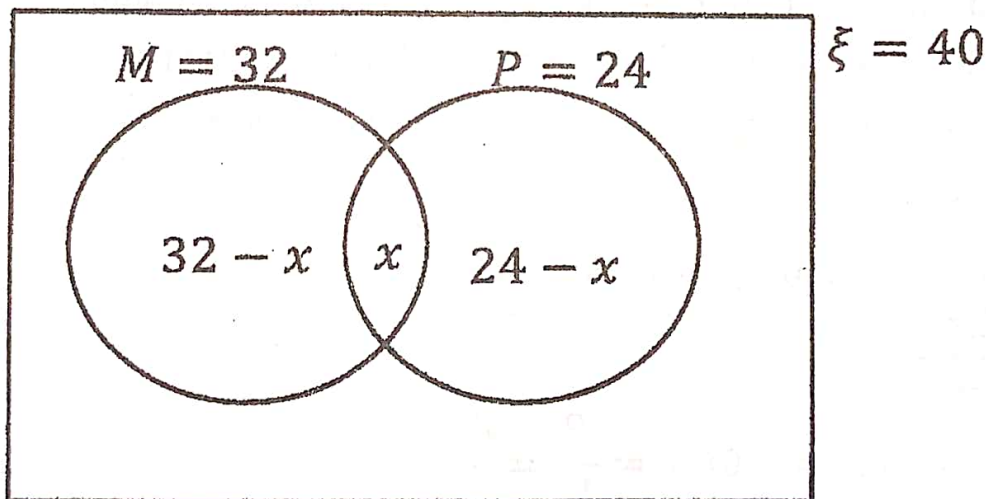
$$\text{Mean } (\bar{x}) = \frac{7+3+14+9+7+8}{6} = 8$$

$$\text{Mean deviation} = \frac{|7-8| + |3-8| + |14-8| + |9-8| + |7-8| + |8-8|}{6}$$

$$= \frac{|-1| + |-5| + |6| + |1| + |-1| + |0|}{6}$$

$$= \frac{1 + 5 + 6 + 1 + 1 + 0}{6} = \frac{14}{6} = 2\frac{1}{3}$$

23. B:



Let those offering both be  $x$

$$32 - x + x + 24 - x = 40$$

$$56 - x = 40$$

$$x = 56 - 40 = 16$$

**Method 2**

$$n(M) + n(P) = n(M \cup P) + n(M \cap P)$$

$$32 + 24 = 40 + n(M \cap P)$$

$$n(M \cap P) = 32 + 24 - 40 = 16$$



$$24. \text{ B: } \frac{9^{2x-1}}{27^{x+1}} = 1, \frac{(3^2)^{2x-1}}{(3^3)^{x+1}} = 1$$

$$\frac{3^{4x-2}}{3^{3x+3}} = 1$$

$$3^{4x-2} = 3^{3x+3}$$

$$4x - 2 = 3x + 3$$

$$4x - 3x = 3 + 2; x = 5$$

$$25. \text{ D: } (-x \times 1) - (2 \times 4x) = (3 \times -5) - (4 \times 3x)$$

$$-x - 8x = -15 - 12x$$

$$-x - 8x + 12x = -15, 3x = -15, x = \frac{-15}{3} = -5$$

$$26. \text{ C: } L \propto \frac{1}{W}, L = \frac{K}{W}$$

$$K = LW = 1.5 \times 20 = 30$$

$$27. \text{ A: } L = \frac{\theta}{360^\circ} \times 2\pi r$$

$$= \frac{30}{360^\circ} \times 2 \times \frac{22}{7} \times 21 \text{ cm} = 11 \text{ cm}$$

$$28. \text{ D: The midpoint of points } (x_1, y_1) \text{ and } (x_2, y_2) =$$

$$\left( \frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$$

Then, the midpoint of  $P(-1, 6)$  and  $Q(-3, -4)$

$$= \left( \frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$$

Then, the midpoint of  $P(-1, 6)$  and  $Q(-3, -4) =$

$$\left( \frac{-1+(-3)}{2}, \frac{6+(-4)}{2} \right) = \left( \frac{-4}{2}, \frac{2}{2} \right) = (-2, 1)$$

The midpoint of  $Q(-3, -4)$  and  $R(1, -4) =$

$$\left( \frac{-3+1}{2}, \frac{-4+(-4)}{2} \right) = \left( -\frac{2}{2}, -\frac{8}{2} \right) = (-1, -4)$$

29. **C:** Volume ( $V$ ) of hemisphere  $= \frac{2}{3}\pi R^3$

$$\text{Then, } \frac{dV}{dR} = 3 \times \frac{2}{3}\pi R^2 = 2\pi R^2$$

$$\text{So, when } R = 2, \frac{dV}{dR} = 2\pi \times 2^2 = 8\pi$$

30. **B:** We are to divide using the traditional long-division approach

$$\begin{array}{r} 3x - 5 \\ 2x-1 \overline{) \sqrt{6x^2 - 13x + 5}} \\ \underline{6x^2 - 3x} \phantom{+ 5} \\ -10x + 5 \\ \underline{-10x + 5} \\ \phantom{-10x + 5} \phantom{+ 5} \\ \phantom{-10x + 5} \phantom{+ 5} \phantom{+ 5} \end{array}$$

Thus, the quotient is  $3x - 5$

31. **C:** If  $\sin \theta = \frac{1}{2}$ ;  $\theta = \sin^{-1}(1/2) = 30^\circ$ . However, we are given  $\sin \theta = -\frac{1}{2}$ . We have to recall that sine is only negative in the 3<sup>rd</sup> and the 4<sup>th</sup> quadrants

$$\text{Thus, } \theta = (180 + 30)^\circ \text{ or } (360 - 30)^\circ = 210^\circ \text{ or } 330^\circ$$

32. D:  ${}^{n+1}C_3 = 4({}^nC_3)$

$$\frac{(n+1)(n)!}{[(n+1)-3]!3!} = 4 \times \frac{n!}{(n-3)!3!}$$

$$\frac{(n+1)(n)!}{(n-2)!3!} = 4 \times \frac{n!}{(n-3)!3!}$$

$$\frac{(n+1) \times (n)!}{(n-2) \times (n-3)! \times 3!} = 4 \times \frac{n!}{(n-3)! \times 3!}$$

$$\frac{n+1}{n-2} = 4$$

$$n+1 = 4(n-2)$$

$$n+1 = 4n-8$$

$$n-4n = -8-1$$

$$-3n = -9, n = \frac{-9}{-3} = 3$$

33. A:  $x * y = \frac{x^2 - y^2}{2xy}$

$$\text{Then, } -5 * 3 = \frac{(-5)^2 - 3^2}{2(-5)(3)} = \frac{25-9}{-30} = \frac{16}{-30} = \frac{-8}{15}$$

34. C:  $\frac{\tan 60^\circ - \tan 30^\circ}{\tan 60^\circ + \tan 30^\circ} = \frac{\sqrt{3} - \frac{1}{\sqrt{3}}}{\sqrt{3} + \frac{1}{\sqrt{3}}}$

$$= \frac{3-1}{\sqrt{3}} \div \frac{3+1}{\sqrt{3}}$$

$$= \frac{2}{\sqrt{3}} \div \frac{4}{\sqrt{3}}$$

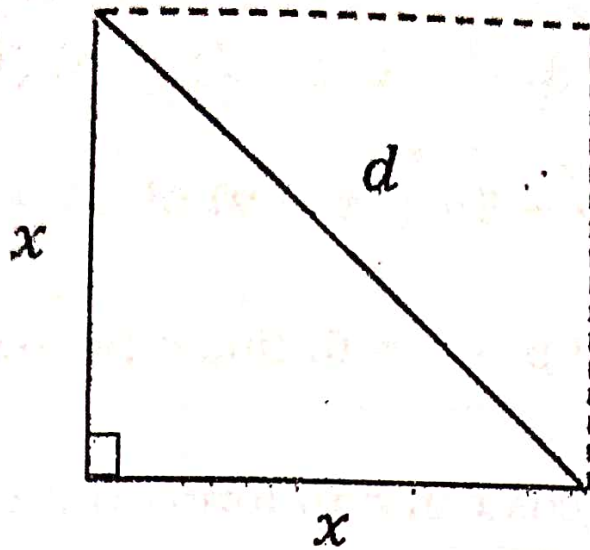
$$= \frac{2}{\sqrt{3}} \times \frac{\sqrt{3}}{4}$$

$$= \frac{2}{4} = \frac{1}{2}$$

35. B: Let the length of each side be  $x$  and the length of the diagonal,  $d$

$$x^2 = 144, x = \sqrt{144} = 12 \text{ cm}$$





Using pythagoras' theorem

$$d^2 = x^2 + x^2$$

$$= 12^2 + 12^2 = 144 + 144 = 288$$

$$d = \sqrt{288} = \sqrt{2 \times 144} = 12 = \sqrt{2}cm$$

36. B:  $\log_{x^{1/2}} 64 = 3,$

$$\left(x^{\frac{1}{2}}\right)^3 = 64$$

$$\left(x^{\frac{1}{2}}\right)^3 = 4^3$$

$$x^{\frac{1}{2}} = 4$$

*Square both sides*

$$\left(x^{\frac{1}{2}}\right)^2 = 4^2$$

$$x = 16$$

37. A: Using difference of two squares  $(4x + 3y)^2 - (3x - 2y)^2 = (4x + 3y + 3x - 2y)(4x + 3y) - (3x - 2y)$

$$= (7x + y)(4x + 3y - 3x + 2y)$$

$$= (7x + y)(x + 5y) = (x + 5y)(7x + y)$$

$$38. \text{ D: } T_n = a + (n - 1)d$$

$$\text{So, } T_5 = a + 4d, T_{11} = a + 10d$$

Since the 5<sup>th</sup> and the 11<sup>th</sup> terms have been quoted as 24 and 96 respectively, we have:

$$a + 4d = 24 \dots (i)$$

$$a + 10d = 96 \dots (ii)$$

Subtract (i) from (ii)

$$10d - 4d = 96 - 24$$

$$6d = 72$$

$$d = 12$$

Put  $d = 12$  into (i)

$$a + 4(12) = 24$$

$$a + 48 = 24$$

$$a = 24 - 48 = -24$$

39. **D:** To get the remainder, we may use the long division or the remainder's theorem

**Method 1 (Long division)**

$$\begin{array}{r}
 2x^2 - 17x^2 + 59 \\
 \hline
 x+3 \overline{) 2x^3 - 11x^2 + 8x - 1} \\
 \underline{2x^3 + 6x^2} \phantom{+ 8x - 1} \\
 -17x^2 + 8x \phantom{- 1} \\
 \underline{-17x^2 - 51x} \phantom{- 1} \\
 59x - 1 \\
 \underline{59x + 177} \\
 -178
 \end{array}$$

**Method 2 (Remainder's Theorem)**

$$f(x) = 2x^3 - 11x^2 + 8x - 1$$

Remainder (R) when  $f(x)$  is divided by  $x + 3 = f(-3)$

$$\text{Then, } R = f(-3) = 2(-3)^3 - 11(-3)^2 + 8(-3) - 1$$

$$\begin{aligned}
 &= 2(-27) - 11(9) + 8(-3) - 1 \\
 &= -54 - 99 - 24 - 1 = -178
 \end{aligned}$$

40. **B:** Only one of them living up to 80 years means 'the husband lives up to 80-yrs, the wife doesn't OR 'the wife lives up to 80yrs, the husband doesn't. the probability that the husband will not live up to 80 yrs  $= 1 - \frac{2}{3} = \frac{1}{3}$ .

The probability that the wife doesn't live up to 80 yrs  $= 1 - \frac{3}{5} = \frac{2}{5}$

Thus, the required probability

$$\begin{aligned}
 &= (P_{husby\ lives} \times P_{wify\ doesn't}) + (P_{wify\ lives} \times P_{husby\ doesn't}) \\
 &= \left(\frac{2}{3} \times \frac{2}{5}\right) + \left(\frac{3}{5} \times \frac{1}{3}\right) = \frac{4}{15} + \frac{3}{15} = \frac{7}{15}
 \end{aligned}$$